# Recursive Access Class Barring for Machine Type Communications with PUSCH Resource Constraints 

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#### Abstract

In massive cellular internet-of-things (IoT) networks, periodic and sporadic traffic may be well processed, while bursty traffic may cause an unexpected network congestion or overload problem. Thus, in this paper, we propose a generalized random access (RA) control mechanism considering whole steps of RA procedure and available resources at each step for handling bursty traffic in cellular IoT networks. The proposed RA control mechanism mainly consists of the estimation method for the number of backlogged nodes and the computation method for access class barring (ACB) factors. Through extensive computer simulations, the proposed RA control mechanism shows the enhanced performance in terms of the total service time, the average access delay, and the energy efficiency, compared to the conventional RA control mechanism, which only focuses on controlling preamble transmissions at the first step of the RA procedure.


Index Terms-Machine type communications, internet-ofthings, random access, access class barring, preamble, PUSCH

## I. InTRODUCTION

In emerging cellular internet-of-things (IoT) and machine-to-machine ( M 2 M ) communication networks, a massive number of IoT devices generate various types of data traffic [1]. Especially, periodic and sporadic traffic may be well processed in the typical cellular networks such as LTE and LTE-advanced, while bursty traffic may cause an unexpected network congestion/overload and resource shortage problems [2]. Initial random access (RA) procedure may be the first bottleneck since a huge number of idle nodes wake up and attempt RAs simultaneously for switching to the radio resource control (RRC) connected mode [3]. Hence, various techniques have been proposed to resolve the RA overload problem in the cellular IoT systems [4].

The access class barring (ACB) scheme has been known as a leading technology in push-based access control schemes, which is very effective to control bursty traffic in cellular IoT networks [5]. In general, the ACB schemes inherently control the number of simultaneous RA attempts with an ACB factor ranging between 0 and 1 [6], [7]. In particular, Jin et al. [7] proposed the recursive Pseudo-Bayesian ACB method for bursty traffic, which dynamically updates the backlog estimation based on the number of available preambles and
the number of unused preambles, and controls the number of simultaneous RA attempts by ACB factors. Even though the backlog estimation is quite well matched with the actual number of backlogged devices, it fragmentarily considers the first step of the RA procedure associated with available preambles in order to control bursty RA traffic.

Likewise, ACB schemes in literature have little or not focused on available random access response (RAR) messages at the second step of the RA procedure and available physical uplink shared channel (PUSCH) resources at third step of the RA procedure, but available preambles at the first step of the RA procedure. Even though a larger number of nodes successfully transmit their preambles without collisions at the first step of the RA procedure, there may be a much severer bottleneck at the second and third steps of the RA procedure due to lack of RAR messages and PUSCH resources [8]-[10].

Hence, in this paper, we propose a generalized RA control mechanism considering whole steps of the RA procedure and available resources at each step for handling bursty traffic in cellular IoT networks. The proposed RA control mechanism mainly consists of the computation method for ACB factors and the estimation method for the number of backlogged nodes. Through extensive computer simulations, we compare the performance of the proposed RA control mechanism with that of the conventional RA control mechanism [7], which only focuses on controlling preamble transmissions at the first step of the RA procedure, in terms of the total service time, the average RA delay, and the energy efficiency.

## II. System Model

In the proposed RA control mechanism, we will consider following parameters:

- $M$ : the number of available preambles at step 1.
- $Q$ : the number of available RAR messages at step 2 .
- $U$ : the number of available PUSCH resources at step 3.
- $K$ : the number of allocable PUSCH resources at step 2.
- $\nu_{i}$ : the est. number of backlogged nodes for the $i$ th slot.
- $p_{i}: \mathrm{ACB}$ factor for the $i$ th slot.

In particular, a single RAR message can convey a single uplink grant for PUSCH resource [11], the number of allocable

PUSCH resources at step 2 is obtained as

$$
\begin{equation*}
K=\min \{Q, U\} \tag{1}
\end{equation*}
$$

Thus, we only consider the number of allocable PUSCH resources $K$ instead of $Q$ and $U$ throughput the paper.

## A. Overall Steps

The overall steps include the access check (Step 0) and the four-step contention-based RA procedure (Step 1-4) [12]:
(Step 0) Access check: The ACB factor of the $i$ th PRACH $p_{i} \in[0,1]$ is notified by the eNodeB before the beginning of the $i$ th PRACH slot. Then, each of active nodes generates a random number $q \in[0,1]$ and compares $q$ with $p_{i}$. If $q \leq p_{i}$, the active node attempts an RA on the $i$ th PRACH. Otherwise, it defers an RA attempt to the $(i+1)$ th PRACH.
(Step 1) Preamble transmission and detection: Each of active nodes with $q \leq p_{i}$ randomly selects a single preamble out of $M$ available preambles, and then transmits the selected preamble on the $i$ th PRACH slot. If two or more nodes select the identical preamble, a preamble collision occurs. However, during the preamble detection, the eNodeB can only identify whether a specific preamble is detected or not, but it cannot recognize that the detected preamble is transmitted by a single node or multiple nodes.
(Step 2) Random access response: The eNodeB sends RAR messages, each of which conveys the identity of detected preamble, timing advance information, and an uplink resource grant for RA-step 3. If the number of allocable PUSCH resources $K$ is smaller than the number of detected preambles, it randomly chooses $K$ detected preambles, and gives PUSCH resource grants only to them via RAR messages.
(Step 3) Uplink data transmission: Using the PUSCH resource informed by the RAR message, the corresponding node transmits the uplink data. If two or more nodes use the identical preamble and the corresponding RAR message, all relevant nodes transmit their own data on the same PUSCH resource, which makes the eNodeB fails to decode the data. However, from this failed data decoding, a preamble collision can be recognized.
(Step 4) Contention resolution $\&$ ACK: If the eNodeB successfully decodes the data transmitted by a single node, it sends back an ACK message including the node ID obtained from the decoded data, otherwise the eNodeB sends nothing back in order to notify a preamble collision. The nodes which do not obtain their IDs regard the situation as a preamble collision, and they reattempt RA from the step 0 .

## B. Bursty Traffic Model

Each of $N$ machine nodes is assumed to activate at the time $x \in\left(0, T_{\text {act }}\right)$, and the activation time $x$ follows Beta distribution with parameters $\alpha=3$ and $\beta=4$, i.e., [13]

$$
f_{X}(x)=\frac{x^{\alpha-1}\left(T_{\mathrm{act}}-x\right)^{\beta-1}}{\left(T_{\mathrm{act}}\right)^{\alpha+\beta-1} B(\alpha, \beta)}
$$

where $B(\alpha, \beta)=\int_{0}^{1} x^{\alpha-1}(1-x)^{\beta-1} d x$ represents the beta function. Within a bursty time period $T_{\text {act }}, I_{\text {act }}$ slots exist, and
the expected number of newly activated nodes in the $i$ th RA slot is given by $\lambda_{i}=N \int_{t_{i-1}}^{t_{i}} f_{X}(x) d x$ for $i=1,2, \ldots, I_{\text {act }}$.

## III. Proposed Random Access Control Mechanism Using the Optimal ACB factors

We propose an RA control mechanism, which consists of two main parts: the computation algorithm for the optimal ACB factor and the estimation/update algorithm for the number of backlogged nodes. In this Section, we first derive the average numbers of detected, collision-free, and collided preambles at the first step of the RA procedure, and then, we calculate the average access throughput considering available preambles and allocable PUSCH resources. In the last part of the Section, we propose a computation algorithm for the optimal ACB factor in terms of maximizing the average access throughput. In the following Section, the estimation/update algorithm for the number of backlogged nodes will be discussed.

## A. Average numbers of detected, collision-free, and collided preambles at the first step of the RA procedure

At the first step of the RA procedure, each of active nodes which passed the access check, transmits an arbitrary preamble out of $M$ available preambles on PRACH slot, and then, the eNodeB detects preambles. In general, detected preambles is classified into collision-free preambles (transmitted by a single node) and collided preamble (transmitted by two or more nodes), but the eNodeB cannot identify the detected preamble is collision-free or collided at this time. Let first define the following discrete random variables:

- $D$ : the number of detected preambles,
- $S$ : the number of collision-free preambles,
- $C$ : the number of collided preambles,
where $D=S+C$. Let $E_{i}$ denote the event that the $i$ th preamble is detected, and the probability of the event $E_{i}$ with $m$ RA-attempting nodes is written as $\operatorname{Pr}\left\{E_{i} \mid m\right\}$. The joint probability that any $d$ preambles out of $M$ preambles are detected, and, each of any $s$ preambles out of $d$ detected preambles is collision-free with $m$ RA-attempting nodes can be written as

$$
\begin{align*}
& \Psi_{d}^{M}(s \mid m)=\binom{M}{d}\binom{d}{s}\binom{m}{s} s!\frac{(d-s)^{m-s}}{M^{m}} \\
& =\binom{M}{M-d+s}\binom{M-d+s}{s}\binom{m}{s} s!\frac{(d-s)^{m-s}}{M^{m}} \tag{2}
\end{align*}
$$

In Eq. (2), $\binom{M}{d}\binom{d}{s}$ represents the total number of cases that any $d$ preambles are detected out of $M$ preambles, and each of any $s$ preambles out of $d$ detected preambles is collisionfree. $\binom{m}{s} s!(d-s)^{m-s}$ represents the total number of cases that each of $s$ nodes among $m$ RA-attempting nodes transmits an exclusive preamble, and each of remaining $(m-s)$ nodes chooses one of preambles among $(d-s)$ preambles. Then, $\binom{M}{d}\binom{d}{s}\binom{m}{s} s!(d-s)^{m-s}$ is divided by $M^{m}$ representing the total number of cases that each of $m$ RA-attempting nodes selects one of preambles among $M$ preambles.

Based on the inclusion-exclusion principle, $\operatorname{Pr}\left\{E_{1} \cup \cdots \cup\right.$ $\left.E_{L} \mid m\right\}$ can be written by

$$
\begin{align*}
& \operatorname{Pr}\left\{\cup_{i=1}^{L} E_{i} \mid m\right\}=\sum_{i=1}^{L}(-1)^{i+1} \sum_{j=0}^{i} \Psi_{i}^{L}(j \mid m) \\
& =\sum_{i=1}^{L} \sum_{j=0}^{i}(-1)^{i+1}\binom{L}{i}\binom{i}{j}\binom{m}{j} j!\frac{(L-i)^{m-j}}{L^{m}}, \tag{3}
\end{align*}
$$

where $\operatorname{Pr}\left\{\cup_{i=1}^{L} E_{i} \mid m\right\}$ denotes the probability that at least one preamble among $L$ preambles is detected with $m$ RA-attempting nodes. The complementary probability to $\operatorname{Pr}\left\{\cup_{i=1}^{L} E_{i} \mid m\right\}$ is written as

$$
\begin{align*}
& \Phi(L \mid m)=\operatorname{Pr}\left\{\cap_{i=1}^{L} E_{i} \mid m\right\}=1-\operatorname{Pr}\left\{\cup_{i=1}^{L} E_{i} \mid m\right\} \\
& =1-\sum_{i=1}^{L} \sum_{j=0}^{i}(-1)^{i+1}\binom{L}{i}\binom{i}{j}\binom{m}{j} j!\frac{(L-i)^{m-j}}{L^{m}} \\
& =\sum_{i=0}^{L} \sum_{j=0}^{i}(-1)^{i}\binom{L}{i}\binom{i}{j}\binom{m}{j} j!\frac{(L-i)^{m-j}}{L^{m}}, \tag{4}
\end{align*}
$$

which represents the probability that all $L$ preambles are detected with $m$ RA-attempting nodes. Using $\Psi_{d}^{M}(s \mid m)$ and $\Phi(L \mid m)$, we obtain

$$
\begin{align*}
& \operatorname{Pr}\{D=d, S=s \mid m\}=\Psi_{d}^{M}(s \mid m) \Phi(d-s \mid m-s) \\
& =\sum_{i=0}^{d-s} \sum_{j=0}^{i}\binom{M}{d}\binom{d}{s}\binom{m}{s} s!\frac{(d-s)^{m-s}}{M^{m}} \times \\
& \quad(-1)^{i}\binom{d-s}{i}\binom{i}{j}\binom{m-s}{j} j!\frac{(d-s-i)^{m-s-j}}{(d-s)^{m-s}} . \tag{5}
\end{align*}
$$

Now, we consider following joint probability that $s$ preambles out of $d$ detected preambles are collision-free when $m$ of $n$ backlogged nodes simultaneously attempt RAs:

$$
\begin{equation*}
\operatorname{Pr}\{d, s, n, m \mid p, \nu\}=\operatorname{Pr}\{d, s \mid m\} \underbrace{\operatorname{Pr}\{m \mid p, n\}}_{\mathbb{B}_{m}^{n}(p)} \underbrace{\operatorname{Pr}\{n \mid \nu\}}_{\mathbb{P}(n \mid \nu)}, \tag{6}
\end{equation*}
$$

where we assume that $\mathbb{P}(n \mid \nu)=\frac{\nu^{n}}{n!} e^{-\nu}$ follows the Poisson distribution with mean of $\nu$, and $\mathbb{B}_{m}^{n}(p)=\binom{n}{m} p^{m}(1-p)^{(n-m)}$ represents Binomial distribution with ACB factor of $p$. By summing up with regard to $n$ and $m$, we have the joint probability that $s$ preambles out of $d$ detected preambles are collision-free with given $p$ and $\nu$

$$
\begin{align*}
& \operatorname{Pr}\{D=d, S=s \mid p, \nu\}=\sum_{n=0}^{\infty} \sum_{m=0}^{n} \operatorname{Pr}\{d, s, n, m \mid p, \nu\} \\
& =\binom{M}{d}\binom{d}{s} e^{-p \nu}\left(\frac{p \nu}{M}\right)^{s}\left(-1-\frac{p \nu}{M}+e^{\frac{p \nu}{M}}\right)^{d-s} \tag{7}
\end{align*}
$$

We omit the detailed derivation of (7) due to space limit. From
(7), we obtain $\operatorname{Pr}\{D=d \mid p, \nu\}$ and $\operatorname{Pr}\{S=s \mid p, \nu\}$ as follows:

$$
\begin{align*}
& \operatorname{Pr}\{D=d \mid p, \nu\}=\sum_{s=0}^{d} \operatorname{Pr}\{D=d, S=s \mid p, \nu\} \\
& =\binom{M}{d} e^{-p \nu}\left(-1+e^{\frac{p \nu}{M}}\right)^{d}  \tag{8}\\
& \operatorname{Pr}\{S=s \mid p, \nu\}=\sum_{d=0}^{M} \operatorname{Pr}\{D=d, S=s \mid p, \nu\} \\
& =\binom{M}{s} e^{-p \nu}\left(\frac{p \nu}{M}\right)^{s}\left(-\frac{p \nu}{M}+e^{\frac{p \nu}{M}}\right)^{M-s} \tag{9}
\end{align*}
$$

In addition, the number of collided preambles $C$ is easily obtained by $C=D-S$, and thus, we have $\operatorname{Pr}\{C=c, S=$ $s \mid p, \nu\}$ by substituting $s+c$ for $d$ in Eq. (7). As a result, $\operatorname{Pr}\{C=c \mid p, \nu\}$ is obtained by,

$$
\begin{align*}
& \operatorname{Pr}\{C=c \mid p, \nu\}=\sum_{s=0}^{M-c} \operatorname{Pr}\{C=c, S=s \mid p, \nu\} \\
& =\binom{M}{c} e^{-p \nu}\left(-1-\frac{p \nu}{M}+e^{\frac{p \nu}{M}}\right)^{c}\left(1+\frac{p \nu}{M}\right)^{M-c} \tag{10}
\end{align*}
$$

Next, the expected values of $D, S$ and $C$ are calculated with given $p$ and $\nu$, respectively, by

$$
\begin{align*}
& \mathbb{E}[D \mid p, \nu]=\sum_{d=0}^{M} d\binom{M}{d} e^{-p \nu}\left(-1+e^{\frac{p \nu}{M}}\right)^{d} \\
& =M\left(1-e^{-\frac{p \nu}{M}}\right)  \tag{11}\\
& \mathbb{E}[S \mid p, \nu]=\sum_{s=0}^{M} s\binom{M}{s}\left(\frac{p \nu}{M}\right)^{s} e^{-p \nu}\left(-\frac{p \nu}{M}+e^{\frac{p \nu}{M}}\right)^{M-s} \\
& =p \nu e^{-\frac{p \nu}{M}}  \tag{12}\\
& \mathbb{E}[C \mid p, \nu]=M\left(-1-\frac{p \nu}{M}+e^{\frac{p \nu}{M}}\right) e^{-\frac{p \nu}{M}} \tag{13}
\end{align*}
$$

B. Average Access Throughput and Optimal ACB factor in terms of average access throughput maximization

After detecting preambles, the eNodeB allocates PUSCH resources to detected preambles via RAR messages. When the allocable PUSCH resources are insufficient for the detected preambles, some nodes cannot receive PUSCH resource grants at the second step of the RA procedure. Hence, the successful access is achieved only in case that a node transmits an exclusive preamble and obtain a PUSCH resource grant. Let $T$ denote the access throughput, i.e, the number of nodes that succeed in random access. For the calculation of the access throughput, we need to consider two cases: $D \leq K$ and $D>K$. Although $\operatorname{Pr}\{T=t \mid p, \nu\}$ depend on $p$ and $\nu$, we suppress them for notational simplicity. Thus, we can write $\operatorname{Pr}\{T=t\}$ as follows

$$
\begin{align*}
& \operatorname{Pr}\{T=t\}=\operatorname{Pr}\{T=t, D \leq K\}+\operatorname{Pr}\{T=t, D>K\} \\
& =\operatorname{Pr}\{D \leq K, S=t\}+\operatorname{Pr}\{D>K, S \geq t\} \mathbb{B}_{t}^{s}\left(\frac{K}{d}\right), \tag{14}
\end{align*}
$$

where $\mathbb{B}_{t}^{s}\left(\frac{K}{d}\right)=\binom{s}{t}\left(\frac{K}{d}\right)^{t}\left(1-\frac{K}{d}\right)^{s-t}$. Then, the expected value of $T$ is calculated by

$$
\begin{align*}
\mathbb{E}[T \mid p, \nu] & =\sum_{t=0}^{K} \sum_{d=0}^{K} t \operatorname{Pr}\{D=d, S=t\}+ \\
& \sum_{t=0}^{K} \sum_{d=K+1}^{M} \sum_{s=t}^{d} t \operatorname{Pr}\{D=d, S=s\} \mathbb{B}_{t}^{s}\left(\frac{K}{d}\right) \tag{15}
\end{align*}
$$

We should find the optimal $p$ to maximize the average access throughput by setting the following optimization problem:

$$
\begin{equation*}
p^{*}=\underset{0 \leq p \leq 1}{\arg \max } \mathbb{E}[T \mid p, \nu] \tag{16}
\end{equation*}
$$

Especially, when $K=M, \mathbb{E}[T \mid p, \nu]=\mathbb{E}[S \mid p, \nu]$, thus, the optimal $p^{*}$ is found by taking the first derivative of $\mathbb{E}[S \mid p, \nu]=$ $p \nu e^{-\frac{p \nu}{M}}$ with respect to $p$ and letting it to be equal to 0 , i.e,

$$
\begin{equation*}
p^{*}=\frac{M}{\nu}, \tag{17}
\end{equation*}
$$

which is equivalent to the results shown in [7], where the constraint of PUSCH resources is ignored. Note that since $\mathbb{E}[S \mid p, \nu]$ is a concave function due to $d^{2} \mathbb{E}[S \mid p, \nu] / d p^{2}<0$, ACB factors should be always set to be equal to or less than $p^{*}=\frac{M}{\nu}$. However, when $K<M$, it is difficult to find the closed-form expression for the optimal value of $p^{*}$. Hence, we can approximate $\mathbb{E}[T \mid p, \nu]$ as

$$
\begin{equation*}
\mathbb{E}[T \mid p, \nu] \approx \mathbb{E}[S \mid p, \nu] \cdot \min \left\{1, \frac{K}{\mathbb{E}[D \mid p, \nu]}\right\} \tag{18}
\end{equation*}
$$

In case of $\mathbb{E}\left[D \mid p^{*}, \nu\right] \leq K<M$, the ACB factor can be the same as $p^{*}$. However, in opposite case of $\mathbb{E}\left[D \mid p^{*}, \nu\right]>K$, $\mathbb{E}[T \mid p, \nu]$ increases until $\mathbb{E}[D \mid p, \nu]=K$, then, it decreases since $\mathbb{E}[S \mid p, \nu]<\mathbb{E}[D \mid p, \nu]$ and $\mathbb{E}[D \mid p, \nu]$ is an increasing function. It implies that $p$ should satisfy $\mathbb{E}[D \mid p, \nu]=K$. In other words, we need to control all of RA-attempting nodes to obtain PUSCH resource grants in order to maximize the access throughput. As a result, we have the following approximated ACB factor

$$
\begin{equation*}
\tilde{p}=\min \left\{1,-\ln \left(1-\frac{K}{M}\right)\right\} \frac{M}{\nu} \tag{19}
\end{equation*}
$$

Fig. 1 shows an example of average access throughput when $K=20$ and $\nu=80$. More specifically, on average, for $p \leq \tilde{p}$, the eNodeB can allocate PUSCH resources to all of detected preambles, and then, $\mathbb{E}[S \mid p, \nu]$ nodes can succeed in RA. On the other hand, for $p>\tilde{p}$, the eNodeB only chooses $K$ preambles among $\mathbb{E}[D \mid p, \nu]$ preambles and allocates PUSCH resources to them, and then, on average $K \cdot \mathbb{E}[S \mid p, \nu] / \mathbb{E}[D \mid p, \nu]$ nodes can succeed in RA on a single PRACH slot. It is worth noting that $\mathbb{E}\left[T \mid p^{*}, \nu\right]-\mathbb{E}[T \mid \tilde{p}, \nu]$ is negligible. Algorithm 1 summarizes the computation algorithm for the ACB factors. Estimation method of $\nu$ will be explained in the next Section.


Fig. 1. The number of detected preambles, the exact access throughput, and the approximated access throughput when $K=20$ and $\nu=80$.

```
Algorithm 1 Computation algorithm for the ACB factor
    Given \(M\) preambles and \(K\) allocable PUSCH resources.
    Estimate the number of backlogged nodes \(\nu\).
    if \(K=M\) then
        \(p=p^{*}=\min \left\{1, \frac{M}{\nu}\right\}\)
    else
        \(p=\tilde{p}=\min \left\{1, \min \left\{1,-\ln \left(1-\frac{K}{M}\right)\right\} \frac{M}{\nu}\right\}\).
    end if
```


## IV. Bayesian Estimation and update of $\nu$

Before generating an ACB factor, the number of backlogged nodes $\nu$ should be estimated. In order to estimate $\nu$, we utilize the Bayesian estimation and update algorithm based on the observations of the number of undetected (idle) preambles [7]. After nodes attempt RAs by transmitting preamble signals according to the ACB factor $p$, which is computed based on the estimated value of $\nu, K$, and $M$, the eNodeB can exactly identify the number of undetected preambles $r$ during the preamble detection procedure. It is notable that at the first step, the eNodeB cannot distinguish the collision or not for the detected preambles. Given this information $r$, we can correct the estimated value of $\nu$ by the estimation offset $\Delta \nu=\mathbb{E}[n \mid r, p, \nu]-\nu$, where $\mathbb{E}[n \mid r, p, \nu]$ is written as [7]

$$
\begin{align*}
\mathbb{E}[n \mid r, p, \nu] & =\frac{\sum_{n=0}^{\infty} n \operatorname{Pr}\{n, r \mid p, \nu\}}{\operatorname{Pr}\{r \mid p, \nu\}} \\
& =\nu\left[(1-p)+\frac{p\left(1-\frac{r}{M}\right)}{\left(1-e^{-\frac{p \nu}{M}}\right)}\right] \tag{20}
\end{align*}
$$

```
Algorithm 2 Estimation and update algorithm of \(\nu\)
    Initialize \(\nu_{0}=M\) and \(k_{0}=0\).
    if \(p_{i-1}=p_{i-1}^{*}\) then
        \(\Delta=\frac{M e^{-1}-r}{1-e^{-1}}\)
    else if \(p_{i-1} \stackrel{e^{-}}{=} \tilde{p}_{i-1}\) then
        \(\Delta=-\ln \left(1-\frac{K}{M}\right)\left(\frac{M}{K}\right)(M-K-r)\)
    end if
    \(\nu_{i-1}=\nu_{i-1}+\Delta \nu \quad \triangleright\) Update of \(\nu\)
    if \(\Delta \nu>0\) then
        \(k_{i}=k_{i-1}+1\) and \(\nu_{i-1}=\nu_{i-1}+k_{i} \cdot \Delta \nu \quad \triangleright\) Boosting
    else
        \(k_{i}=0\) and \(\nu_{i-1}=\nu_{i-1}\)
    end if
    \(\nu_{i}=\max \left(1, \nu_{i-1}-c_{i-1}\right) \quad \triangleright\) New estimation of \(\nu\) for slot \(i\)
```

TABLE I
Simulation parameters and values

| Parameters | Values |
| :---: | :---: |
| The number of preambles, $M$ | 64 |
| The number of allocable PUSCH resources, $K$ | $20-64$ |
| PRACH interval time, $T_{\text {interval }}$ | 1 ms |
| Activation duration, $T_{\text {act }}$ | 500 ms |
| The number of activation slots, $I_{\text {act }}$ | 500 slots |
| Total number of RA-attempting nodes, $N$ | 10000 |
| Bursty traffic parameters, $\alpha$ and $\beta$ | 3 and 4 |

In particular, we have estimation offsets:

$$
\begin{align*}
& \Delta \nu=\mathbb{E}[n \mid r, p, \nu]-\nu=\nu p\left(\frac{e^{-\frac{p \nu}{M}}-\frac{r}{M}}{1-e^{-\frac{p \nu}{M}}}\right),  \tag{21}\\
& \left.\Delta \nu\right|_{p=p^{*}}=\frac{M e^{-1}-r}{1-e^{-1}}  \tag{22}\\
& \left.\Delta \nu\right|_{p=\tilde{p}}=-\ln \left(1-\frac{K}{M}\right)\left(\frac{M}{K}\right)(M-K-r), \tag{23}
\end{align*}
$$

where $p^{*}=\frac{M}{\nu}$ and $\tilde{p}=-\ln \left(1-\frac{K}{M}\right) \frac{M}{\nu}$.
Algorithm 2 summarizes the estimation and update algorithm of $\nu$. The eNodeB first estimates $\nu_{0}$ as the number of preambles $M$. Following [7], we also introduce a boosting factor $k_{i}$ in order to consider bursty traffic. It implies that when we observe that the traffic continuously increases based on $\Delta \nu>0$, we boost the estimation $\nu$. In line 13, the new estimation of $\nu_{i}$ is calculated by $\nu_{i}=\nu_{i-1}-c_{i-1}$, where $c_{i-1}$ denotes the number of RA success nodes on the $(i-1)$ th PRACH slot.

## V. Numerical Results

Table I summarizes the simulation parameters and values. We assume that PRACH appears every 1 ms , i.e, $T_{\text {interval }}=1$ ms. 10000 machine nodes attempt to connect to the network in a bursty manner through the four-step RA procedure as varying PUSCH resources, $K$, from 20 to 64 . We evaluate the performance of the proposed mechanism in terms of the total service time, the average RA delay, and the energy efficiency, compared to the Jin's RA control scheme [7] and the ideal scheme. The ideal scheme follows the same RA control mechanism as the proposed mechanism, but it is assumed to exactly know the number of backlogged nodes on every slot.


Fig. 2. Estimation for 10000 backlogged machine nodes with $K=20$ and $K=64$.


Fig. 3. Total service time

Fig. 2 shows the Bayesian estimation result for the number of backlogged nodes with sufficient PUSCH resources ( $K=$ 64) and insufficient PUSCH resources ( $K=20$ ), respectively. Even though there exist a gap between the estimation and actual values, the estimation algorithm keeps well track of the actual values especially during the traffic descent period. Due to the backlog estimation errors, the proposed RA control mechanism may show the degraded performance, compared to the ideal mechanism.

Fig. 3 shows the total service time for varying the number of allocable PUSCH resources $K$. The proposed RA control mechanism generates the ACB factors by considering both of the available preambles and the allocable PUSCH resources, while the Jin's scheme only considers the available preambles. Thus, when the PUSCH resources are insufficient compared to the number of detected preambles, e.g., $K<40$, the proposed mechanism takes shorter time for all nodes to complete the RA service, compared to the Jin's scheme. In terms of the total service time, the proposed mechanism shows the similar result to that of the ideal scheme.


Fig. 4. Average random access delay.


Fig. 5. Energy efficient when $E_{S 1}=E_{S 3}$.

Fig. 4 shows the average RA delay for varying the number of allocable PUSCH resources $K$. We can observe a huge difference of average RA delay between the proposed mechanism and the Jin's scheme. In case of $K=20$, the proposed mechanism shows the average RA delay of 168 ms , while the Jin's scheme shows the average RA delay of 268 ms .
Lastly, Fig. 5 shows the energy efficiency for varying the number of allocable PUSCH resources $K$, where we assume that the uplink transmission energy for RA-step 1 and 3 is identical, i.e., $E_{\mathrm{S} 1}=E_{\mathrm{S} 3}$. The energy efficiency of the proposed mechanism decreases from approximately 0.65 to 0.36 , while the energy efficiency of the Jin's scheme increases from approximately 0.25 to 0.36 . The result implies that we should consider the allocable PUSCH resources in order to control massive energy-critical devices for successful RAs.

## VI. Conclusion

In this paper, we proposed an RA control mechanism for massive IoT networks. Specifically, we utilized the Bayesian backlog estimation algorithm to estimate the number of backlogged nodes, and based on this information, the opti-
mized ACB factor is computed by considering the available preambles and the allocable PUSCH resources in order to maximize the access throughput. Through extensive computer simulations, we evaluated the performance of the proposed mechanism in terms of the total service time, the average RA delay, and the energy efficiency. The proposed mechanism outperforms the conventional scheme when the PUSCH resources are insufficient, which implies that RA control should carefully consider not only the available preambles but also the allocable PUSCH resources. Especially, the proposed mechanism shows a higher energy efficiency than the conventional scheme, where the energy efficiency is very important factor for batteryconstrained IoT devices.

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